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*A Formula for expressing the Decrement of Human Life.
By the late DR. THOMAS YOUNG.*

(Continued from p. 353, Vol. VI.)

Characteristics of Mortality.

	Annual Mortality 1 in	Mean Full Term of Life.	Mean Age.
Roman estimate of Ulpian, probably with some deduction for present value	74 + disc ^c	26 (+ disc ^c)
Deparcieux's Tontines, beginning 1689	47·67	94·17	32·5
Halley's Table for Breslau, 1690	33·50	87·15	28·1
Tontine of 1695 (Finlaison)—Males	37·61	83·42	27·25
Females	(43·0)	87·50	29·32
Simpson's Table for London, about 1730	19·2	82·30	25·7
Dupré, in Buffon, about 1750	33·0	85·30	28·67
Northampton Tables, about 1760	25·18	87·39	28·86
Swedish Tables, about 1785	36·12	91·86	31·3
France, before the Revolution (Duvillard)	28·76	86·96	29·0
Finlaison's Tontine and Annuitants, about 1800—			
Males	50·16	93·25	32·0
Females	55·51	100·7	34·6
Finlaison's Chelsea Pensioners	90·0	29·65
Carlisle Tables, about 1810 (Milne)	37·14	95·47	32·6
Returns for all England, 1811	49		

Another mode of easily appreciating the regularity and the analogies of different tables is, to construct a diagram, in the form of a curve, of which the absciss represents the age, and the ordinates the corresponding decrements of life (*see opposite*). The inspection of such a diagram is sufficient to convince us of the great irregularity of the Carlisle Tables of Mortality, which must obviously have been formed, as they confessedly were, from observations on a very limited number of individuals, so that they exhibit a succession of different climacterics, after which the mortality is diminished; while about the age of 74 the curve that represents them towers to an incredible height, affording an expectation of longevity which some of the strongest advocates of those tables have abandoned in their practical applications, since they take their estimate of life, in advanced age, even lower than it is represented in the Northampton Tables.

It appears, therefore, to be highly probable, that the fairest basis for general computations, to be applied throughout Great Britain, may be obtained by a proper combination of the Tables of Northampton (which have been long known and very generally approved) with the Carlisle Tables—corrected, however, in their extravagant values of old lives, by some other documents—and with

the mortality of London, as derived from the parish registers: which, when thus incorporated with tables formed in the country, will be freed from the objections that have been made to the observations of burials in great cities only.

The Carlisle Table agrees in the earlier parts pretty nearly with the observations of Mr. Morgan on the experience of the Equitable Office from 1768 to 1810, as it appears from Mr. Milne's comparison, as well as from the reduction and interpolation of those observations, published by Mr. Gompertz in the *Philosophical Transactions* for 1825; but for correcting the later portions of the Carlisle Table, it may be allowable to employ a subsequent register of the experience of the Equitable Office, so far as it is possible to make any inferences from it with safety.

The numbers of deaths occurring in twenty years, as recorded by Mr. Morgan, might have been made the foundation of a very valuable determination of the mortality occurring in a certain class of persons, if the number of the Equitable Society had become stationary before the commencement of the record; but in order to deduce from it a just estimate of the value of life, it would then be necessary to alter the numbers of deaths at each age, in the inverse proportion of the numbers of the living compared—that is to say, not simply of the sums of the persons admitted under that age, but of the numbers of persons born whom they represent; since, in comparing the joint mortalities of any two lists of persons, we must obviously add together the deaths belonging, not to a given number of persons of various ages, but of a number proportionate to the survivors at the respective ages out of a given number of births: so that in this manner the apparent mortality in the earlier portions of the register would require to be augmented, not only on account of the smaller number of persons who have actually contributed to furnish it, but also on account of the greater proportion that these persons bear to the corresponding number at birth, when compared with the survivors at more advanced ages, who represent a population still more exceeding their own numbers. On the other hand, since the register in question relates only to a limited number of years, immediately following a very rapid increase of the Society, it is evident that the deaths must have occurred at earlier ages than if it had been continued until all the lives had dropped.

Of these three modifications, it may be sufficiently accurate for the present purpose to omit the two latter, as nearly counterbalancing each other, and to augment the earlier numbers in the proportion only of the members of the Society to whom they must necessarily

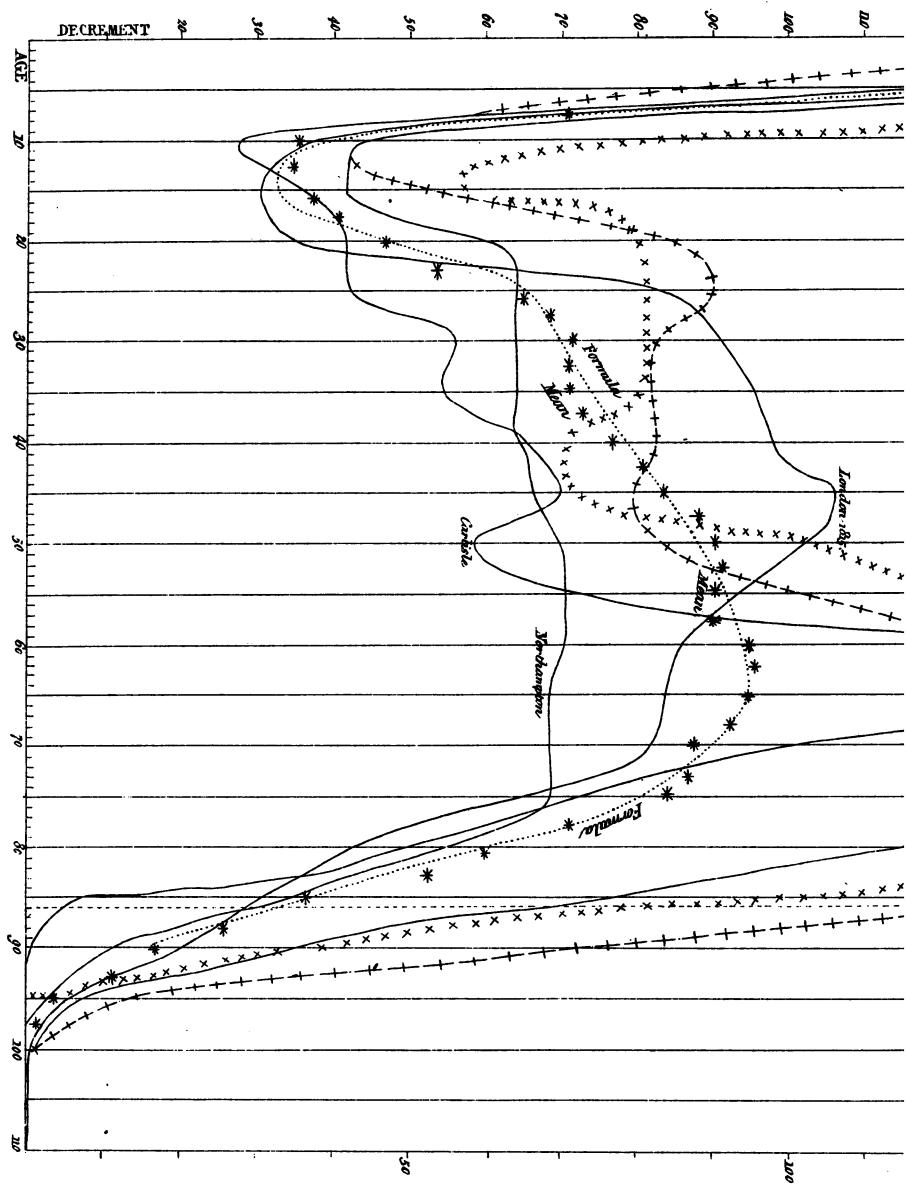
have belonged, supposing that the admissions had taken place about the same ages at all periods ; assuming also the number of survivors at 45 to be in the same proportion to the births as in the Carlisle Table. We may then proceed to take a mean between the mortality thus obtained, with proper interpolations, and the observations at Carlisle, as the second of the three principal bases to be afterwards incorporated with the mortality of Northampton and of London. Further than this, it is impossible to place any great reliance on Mr. Morgan's document, which makes the annual deaths, in "a population exceeding 150,000," not quite 1 in 1,500.

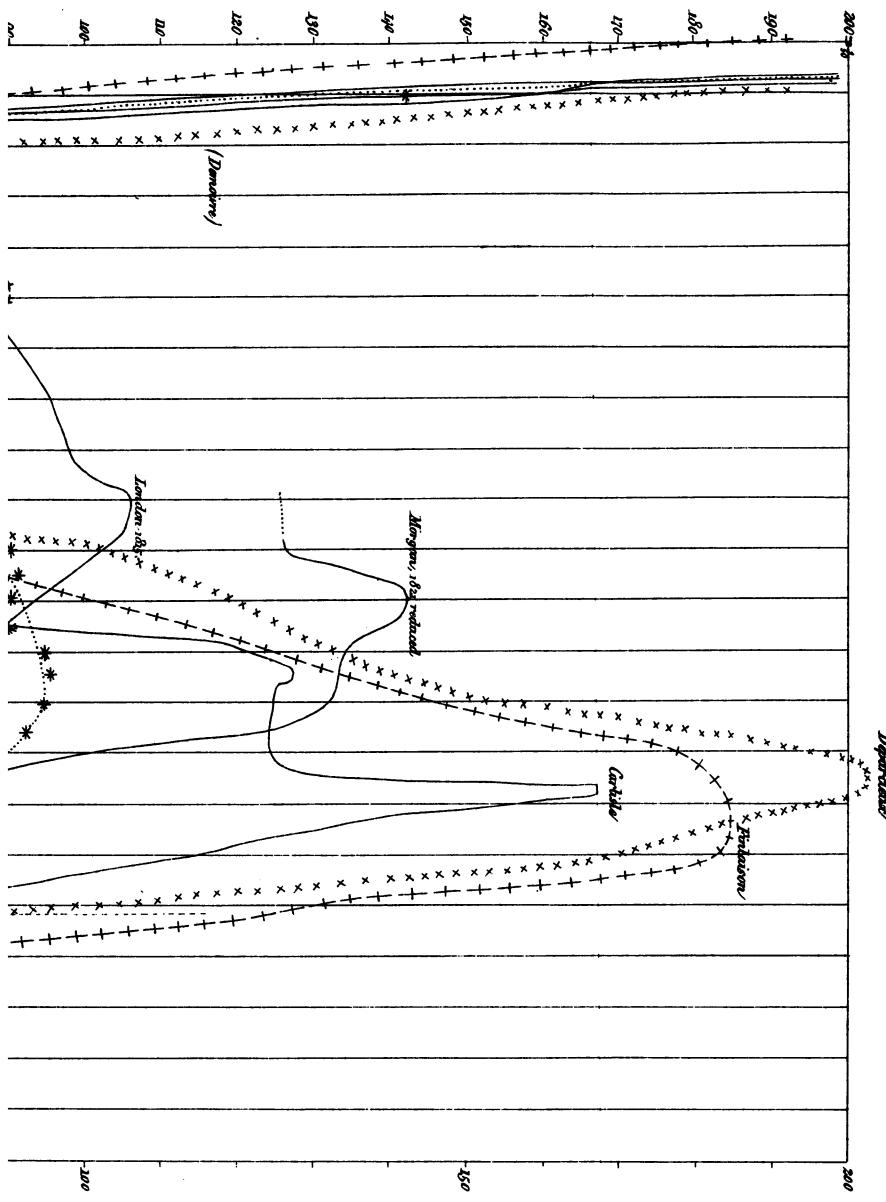
Of the mortality of London, taken for the ten years from 1811 to 1820, it may be observed, that its results bear the internal evidence of greater apparent correctness than either of the other bases, exhibiting a curve less irregular in its flexures, and generally intermediate between the others : it has also the advantage of exhibiting the duration of life as prolonged by the general introduction of vaccination : and when thus incorporated with the registers of two places in the country, each reduced to an equal supposed population, it must probably be sufficiently corrected for the errors that may be attributed to the effect of an afflux of settlers at an early age. The mean obtained in this manner might be employed at once as a standard table, without much inconvenience ; but it still exhibits some minute but obvious irregularities, as an inspection of the line of stars in the diagram will show, principally perhaps from the want of skill or care with which the interpolations have been made by Dr. Price and others. The most effectual of all interpolations for *harmonizing* the various orders of differences is to obtain a formula which shall extend with sufficient accuracy throughout the whole curve. It may be easily believed that it must be extremely difficult to find such an expression, and that its form must be too complicated to be applied to any practical purpose throughout its extent. I have, however, drawn a curve which comes extremely near to the line of stars, and crosses it in ten or twelve different points, by means of the equation

$$y = 368 + 10x - 11 \cdot (156 + 20x - x^2)^{\frac{3}{2}} + \frac{1}{2.85 + 2.05x^2 + 2\left(\frac{x}{10}\right)^6} \\ - 5 \cdot 5\left(\frac{x}{50}\right)^{10} + \frac{5 \cdot 5^2}{4000}\left(\frac{x}{50}\right)^{20} - 5500\left(\frac{x}{100}\right)^{40} :$$

y being the number of deaths among 100,000 persons, in the year that completes the age x .

The terms of this formula have some remarkable relations to the different periods of life. Halley's first approximation was





$y=1,000$, throughout life. De Moivre's arithmetical hypothesis was $y=\frac{100,000}{86}=1163$: but of the present formula the principal foundation, as extending to the whole of life, is $y=368+10x$. In infancy, the term containing the reciprocal of the powers of x has a preponderating value; in youth, the term $-(156+20x-x^2)^{\frac{3}{2}}$, which diminishes the mortality, ends somewhat abruptly at 25, and would be incapable of being employed with safety in algebraical calculations, from its having a negative as well as a positive value. Old age is expressed almost exclusively by the high powers at the end of the formula, which terminate the series with great and increasing rapidity. It is obvious that, for many purposes of calculation, the terms belonging to youth and to old age might be neglected without inconvenience, and that, for the middle portion of life, the terms $368+10x$ alone, with some little modification, might be employed as sufficiently correct; or, certainly, as much nearer to the truth than either the arithmetical or geometrical hypothesis of De Moivre. The relations of the different parts of the formula will be best appreciated from their development in the following tables.

Decremnts of Mortality, computed from the Formula.

Age ($x-1$)	368 + $10x$	$- \cdot 11(156+20x-x^2)^{\frac{3}{2}}$	$+$	Decrement.
			$\frac{1}{2 \cdot 85 + 2 \cdot 05x^2 + 2\left(\frac{x}{10}\right)^6}$	
0	378	-255	+ 20,408	20,531
1	388	241	9,009	9,106
2	398	313	4,695	4,780
3	408	359	2,805	2,854
4	418	386	1,848	1,880
5	428	409	1,322	1,341
6	438	427	968	979
7	448	440	746	752
8	458	447	592	603
9	468	451	477	494
10	478	447	392	423
11	488	440	329	377
12	498	427	278	349
13	508	409	238	337
14	518	386	205	337
15	528	359	178	347
16	538	313	156	381
17	548	291	136	393
18	558	255	119	422
19	568	214	104	458
20	578	174	93	497
21	588	130	82	540
22	598	89	72	581
23	608	51	64	621
24	618	19	57	656
25	628	0	50	678

Decremens of Mortality, computed from the Formula—(continued).

Age ($x-1$)	$368 + 10x$	$+ \frac{1}{2.85 + 2.05x^2 + 2\left(\frac{x}{10}\right)^6}$	$-5.5\left(\frac{x}{50}\right)^{10}$	$+.001\left(\frac{5.5\left(\frac{x}{50}\right)^{10}}{2}\right)^2$	Decre- ment.
26	638	44	682
27	648	39	687
28	658	34	692
29	668	30	698
30	678	27	705
31	688	24	712
32	698	21	719
33	708	18	726
34	718	16	734
35	728	14	742
36	738	13	751
37	748	11	-0	..	759
38	758	10	.4	..	768
39	768	9	.6	..	776
40	778	8	.7	..	785
41	788	8	.9	..	795
42	798	7	1.2	..	804
43	808	6	1.5	..	813
44	818	5	1.9	..	821
45	828	5	2.3	..	831
46	838	4	3.0	..	839
47	848	4	3.9	..	848
48	858	3	4.5	..	857
49	868	3	5.5	..	866
50	878	3	6.7	..	874
51	888	2	8	..	882
52	898	2	10	..	890
53	908	2	12	..	898
54	918	2	14	..	906
55	928	-2	17	..	913
56	938	1	20	..	917
57	948	1	24	..	923
58	958	1	28	..	929
59	968	1	33	..	934
60	978	1	39	+0	938
61	988	1	46	1	942
62	998	1	55	1	943
63	1,008	1	64	1	944
64	1,018	1	75	1	943
65	1,028	..	-88	2	942
66	1,038	..	102	3	939
67	1,048	..	119	4	933
68	1,058	..	137	5	926
69	1,068	..	159	6	915
70	1,078	..	183	8	903
71	1,088	..	211	11	888
72	1,098	..	242	15	871
73	1,108	..	277	19	850
74	1,118	..	317	25	826

Decremens of Mortality, computed from the Formula—(continued).

Age ($x-1$)	368 $+ 10x$	$-5 \cdot 5 \left(\frac{x}{50}\right)^{10}$	$+ .001 \left(\frac{5 \cdot 5 \left(\frac{x}{50}\right)^{10}}{2}\right)^2$	$-5500 \left(\frac{x}{100}\right)^{40} =$	Decre- ment.
75	1,128	359	32	..	801
76	1,138	412	42	..	768
77	1,148	470	55	..	733
78	1,158	532	71	—0	697
79	1,168	604	91	1	654
80	1,178	684	117	1	610
81	1,188	772	145	2	559
82	1,198	872	190	3	513
83	1,208	984	242	6	460
84	1,218	1,108	307	9	403
85	1,228	1,243	386	14	357
86	1,238	1,399	490	22	307
87	1,248	1,567	614	37	258
88	1,258	1,756	771	52	215
89	1,268	1,963	963	86	178
90	1,278	2,192	1,201	139	148
91	1,288	2,444	1,493	212	125
92	1,298	2,713	1,849	333	101
93	1,308	3,032	2,300	496	80
94	1,318	3,371	2,840	734	53
95	1,328	3,744	3,504	1,041	27
96	1,338	4,150	4,306	1,746	3
97	1,348				
98	1,358				
99	1,368				
100	1,378				

*Mean Standard Table of the Decremens of Life in Great Britain,
1824.*

Age.	Decre- ment.	Living.									
0	20,531	100,003	15	347	54,860	30	705	46,527	45	831	35,117
1	9,106	79,472	16	381	54,513	31	712	45,822	46	839	34,286
2	4,780	70,366	17	393	54,132	32	719	45,110	47	848	33,447
3	2,854	65,586	18	422	53,739	33	726	44,391	48	857	32,599
4	1,880	62,732	19	458	53,317	34	734	43,665	49	866	31,742
5	1,341	60,852	20	497	52,859	35	742	42,931	50	874	30,876
6	979	59,511	21	540	52,362	36	751	42,189	51	882	30,002
7	752	58,532	22	581	51,822	37	759	41,438	52	890	29,120
8	603	57,780	23	621	51,241	38	768	40,679	53	898	28,230
9	494	57,177	24	656	50,620	39	776	39,911	54	906	27,332
10	423	56,683	25	678	49,964	40	785	39,135	55	913	26,426
11	377	56,260	26	682	49,286	41	795	38,350	56	917	25,513
12	349	55,883	27	687	48,604	42	804	37,555	57	923	24,596
13	337	55,534	28	692	47,917	43	813	36,751	58	929	23,673
14	337	55,197	29	698	47,225	44	821	35,938	59	934	22,744

*Mean Standard Table of the Decremts of Life in Great Britain, 1824—
(continued).*

Age.	Decremt.	Living.									
60	938	21,810	75	801	8,107	90	164	589	105	1	3
61	942	20,872	76	768	7,306	91	130	425	106	.25	2
62	943	19,930	77	733	6,538	92	87	295	107	.25	1.75
63	944	18,987	78	697	5,805	93	60	208	108	.25	1.50
64	943	18,043	79	654	5,108	94	44	148	109	.25	1.25
65	942	17,100	80	610	4,454	95	31	104	110	.25	1.0
66	939	16,158	81	559	3,844	96	19	73	111	.25	.75
67	933	15,219	82	513	3,285	97	14	54	112	.25	.50
68	926	14,286	83	460	2,772	98	9	40	113	.25	.25
69	915	13,360	84	408	2,312	99	6	31	114	0	0
70	903	12,445	85	357	1,904	100	6	25			
71	888	11,542	86	307	1,547	101	5	19			
72	871	10,654	87	258	1,240	102	5	14			
73	850	9,783	88	215	982	103	4	9			
74	826	8,933	89	178	767	104	2	5			

I shall take this opportunity of endeavouring to demonstrate, in a simple and undeniable manner, the error into which Dr. Price and his followers have fallen, in consequence, as it appears, of their adopting the legal restraints on usury as essential steps in the mathematical calculation of the amount of compound interest. The error has, indeed, of late years been very commonly admitted; but its effects have not been so satisfactorily rectified as could be desired.

In the 66th volume of the *Philosophical Transactions*, for the year 1776, we find a paper of Dr. Price, in which he lays down these theorems, r denoting the interest of £1 for a year, and n the term or number of years during which any annuity will be paid; p the perpetuity, or $\frac{1}{r}$; y the value of an annuity paid yearly, and h half yearly. Then, I., $y=p-\frac{1}{r(1+r)^n}$; and, II., $h=p-\frac{1}{r\left(1+\frac{r}{2}\right)^{2n}}$:

and as examples, taking $r=.04$, and $n=5$, we have $y=4.4518$, and $p=4.4913$.

Now, if we analyze the results thus obtained, by dividing them into the present values of the separate payments, they will stand thus:—

I.	Present value of £1 payable at the end of 1 year	£.961538
"	"	2 years .924556
"	"	3 " .888996
"	"	4 " .854804
"	"	5 " .821927
		£4.451821

				£.
II.	Present value of 10s., payable at the end of half a year			49020
"	"	"	1 year	48058
"	"	"	1½ "	47127
"	"	"	2 "	46192
"	"	"	2½ "	45286
"	"	"	3 "	44398
"	"	"	3½ "	43528
"	"	"	4 "	42674
"	"	"	4½ "	41837
"	"	"	5 "	41018
				<hr/>
				£449138

The present values of 10s. are therefore assumed:

I. At 1 year	·48077	II. ·48058
2 years	·46228	·46192
3 "	·44450	·44398
4 "	·42740	·42674
5 "	·41096	·41018

The latter column exhibiting obviously a larger deduction for discount than the former, so that the rate of interest in the two calculations is by no means the same; although, in the case of $r=·05$, they would respectively represent the highest rate of interest allowed by our laws to be received without a new investment or engagement: but this arbitrary restraint ought certainly not to affect the mathematical consideration of the question.

The difficulty, if any person thinks it such, may be avoided by a mode of investigation which I have lately had occasion to point out:—"An annuity, of which a payment is due on a given day, is more valuable than an annuity purchased on that day, and to commence a year after, by the amount of a year's payment: and *the value of a life annuity becoming payable at any intermediate time between the day of purchase and its first anniversary will be greater than the simple tabular value of the annuity by a sum proportional to the anticipation of the payment;*" the increase of the value being very nearly uniform, when we suppose the anticipation to be gradually increased: this increase of the value comprehending obviously the greater probability as well as the greater proximity of each payment, and proceeding from day to day by very nearly equal increments. Thus, if we wished to purchase an annuity of £100 a year, and its value were £1,000, upon the ordinary supposition of the payments commencing after the end of a year—supposing that we desired to have the first payment made at the end of nine months, and the subsequent payments at annual intervals as usual—we should have to add £25 to the purchase-

money, making it £1,025, at whatever rate of interest the value might have been computed. If we began at six months, £50, and if at three months, £75, must be added to the purchase: it being obvious that an additional £100 would be equivalent to an anticipation of twelve months, or to an immediate payment of a year's annuity.

From this simple and incontestable principle, in which the second differences only are neglected, it is very easy to deduce the values of annuities payable at intervals shorter than a year. An annuity of 1; payable half yearly, is equal to two annuities of $\frac{1}{2}$, the one beginning as usual at the end of the year, the other anticipated by half a year; and the value of this portion is greater than the other by half of one of the payments, that is, by $\frac{1}{4}$: so that "*we may always find the value of a life annuity payable half yearly, by adding a quarter of a year to the tabular value of the same annuity.*"

In a similar manner it is very easily shown, that "*for quarterly payments we must add $\frac{3}{8}$ of a year's value to the computation made on the supposition of annual payments;*" and "*the continual bisection of the interval would at last afford us the addition of half a yearly payment for the value of a daily or hourly payment of a proportional part of the given annuity.*"

"It may also be observed, that when we reckon at 3 per cent. interest, an annuity payable half yearly is the same, throughout the middle of life, that would be granted on the life of a person a year older, if payable annually."

If it is required to ascertain the value of a reversionary annuity payable half yearly or quarterly, the calculation becomes in appearance a little paradoxical; for since the true value of a reversionary annuity for the life of one person, for example, after the death of another, is the difference between the values of two annuities on the single life and the joint lives, and since an equal addition must be made to these values in consideration of the period of payment being shortened, it follows that the reversionary annuity must be of equal value in either form. This conclusion would indeed be strictly true if the periodical times of payment remained unaltered, according to the supposition from which the value of the annuities is deduced; while, in fact, it is usual to grant such an annuity to commence at the first quarterly, half yearly, or annual period after the contingent event—a variation which would have no sensible effect in the case of daily payments, but which lessens the value of reversionary annuities at other periods by that of half a pay-

ment for the given period, reduced to the present time in the same manner as any other sum assured as payable upon the same contingency of survivorship.

The simplicity observable in the progression of the values of annuities, calculated according to the values of lives here supposed, and at 3 per cent. interest, leads us to inquire what would be the exact law of mortality required to make that progression strictly uniform throughout life; and it will appear on investigation, that in order to have the value $24\cdot45 - \frac{1}{4}x$, x being the age of the person, which is nearly true between 20 and 70, the annual mortality must be expressed by $\frac{\cdot03x + \cdot066}{100\cdot8 - x}$: a fraction which at 20 becomes $\frac{1}{121}$; at 40, $\frac{1}{48}$; at 60, $\frac{1}{22}$; and at 80, $\frac{1}{8\cdot4}$. Our table gives respectively $\frac{1}{103}$, $\frac{1}{50}$, $\frac{1}{23}$, and $\frac{1}{7\cdot3}$; the Northampton, $\frac{1}{71}$, $\frac{1}{48}$, $\frac{1}{25}$, and $\frac{1}{7\cdot4}$. Mr. Finlaison's male annuitants, $\frac{1}{87}$, $\frac{1}{73}$, $\frac{1}{32}$, and $\frac{1}{8\cdot3}$.

The healthiness of Mr. Finlaison's annuitants about 40 and 50 is one of the most remarkable features of his table. He observes (p. 58), that out of 10,000 persons at 23, 141 will die in a year, and 141 will die out of the same number at the age of 48; but at the age of 34 there will only die 124. The curve marked by obelisks (+) in the diagram will show the comparative progress of mortality in this system; which, however valuable the data may be, appears to exhibit too many novelties, if not anomalies, to be generally adopted with confidence: while the line of crosses (\times), representing the Tontine of Deparcieux, will serve to show how little difference the lapse of a century has made in the results of these two similar cases.

I shall conclude with a comparison of the climacteric years, as they may be called without impropriety, in which the greatest numbers of adults die, as taken from different tables.

I sincerely hope that these considerations may help to undeceive the too credulous public, who have of late not only received some hints that tend to insinuate the probability of an occasional recurrence of a patriarchal longevity, but who have been required to believe, upon the authority of a most respectable mathematician, that the true and unerring value of life is not to be obtained by taking an average of various decrements, but by adopting the extreme of all conceivable estimates, founded only on a hasty assertion of Mr. Morgan, and unsupported by any detailed report:

an estimate which makes the grand climacteric of mankind in this country, not a paltry fifty-four, or the too much dreaded sixty-three; but no less than EIGHTY-TWO! an age to which nearly one sixth of the survivors at ten are supposed to attain!

Climacterics, or greatest Decremnts.

Berlin, formerly	38	Formula	63	Sweden, 1785	69
London, about 1733 ..	40	Brandenburg	65	Holycross, 1760	70
Paris, formerly	40	Warrington, 1777 ..	65	Deparcieux	73
Stockholm, 1762	42	Norwich, 1765	66	Carlisle	74
London, 1764	43	Montpellier, 1782 ..	67	Ackworth, 1752	75
London, 1815	46	Duvillard, France ..	67	Kersseboom	77
Northampton, 1757 ..	56	Sweden, 1762	68	Finlaison	78
Breslau, 1695	61	Chester, 1776	68	E. O. Mr. B.	82

NOTE.—Some of the tables appended to this paper have been omitted, as devoid of interest at the present day—ED. A. M.

On the Settlement of Losses by Fire under Average Policies. By RICHARD ATKINS, Esq., of the Sun Fire Office.

IT was intended that the short essay descriptive of the existing system of settlements of claims under average fire policies, which appeared in this *Journal* as far back as No. X. (January, 1853), should have been promptly followed by some remarks on the most obvious defects of the system, with a few practical suggestions for their remedy. It is not necessary to explain here the reasons for the delay which has taken place in the fulfilment of that intention; but it is satisfactory to know that the postponement has not been without its use. Many valuable hints have been offered; and although some marked differences of opinion exist as to any proposed remedies, the explanations given of the existing rules are, without any exception, admitted to be correct.

The examples of some remarkably involved claims and their ultimate settlement, which were given in the former article, have been thought serviceable; and suggestions have been made that a considerable number and variety of similar cases of actual settlements should be collected and published, in order that these important precedents may be generally known. It does not, of course, follow, that any previous decisions should come with such a weight of authority as to preclude a fresh discussion on many of these doubtful points; but in the absence of any fixed or acknowledged law it is unquestionably of some value to know what judgments